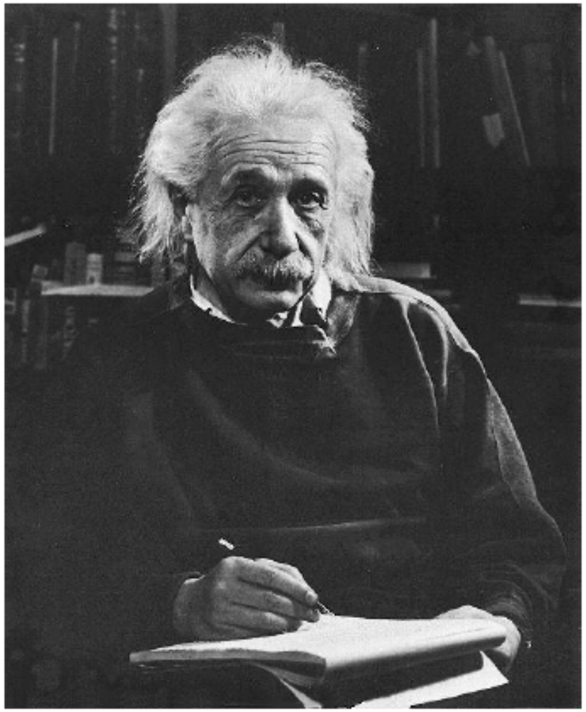
Chapt1 Theory of Special Relativity

- Fundamental postulate
- Michelson interferometer
- Lorentz transformation
- Time dilation
- Length contraction
- The twin paradox
- Adding velocities
- Momentum and energy
- Examples of SR

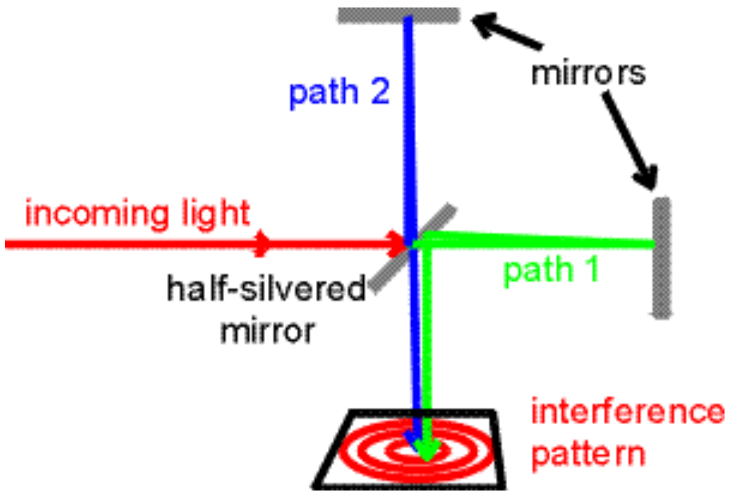
1. The laws of physics are independent of an observer's reference frame

- Mechanics,
- Optics
- Electromagnetics
- Physics laws are convariant in all IFRs

2. The speed of light, ***c***, is the same in all reference frames(IRF).

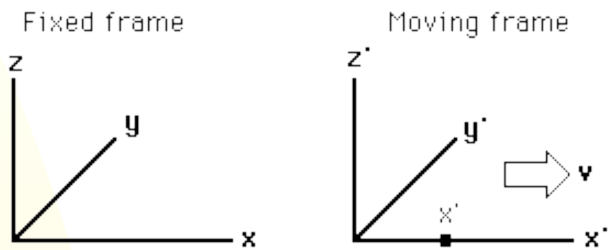


Michelson Morley Experiment



proving the non-existence of the ether.

Lorentz transformation



$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

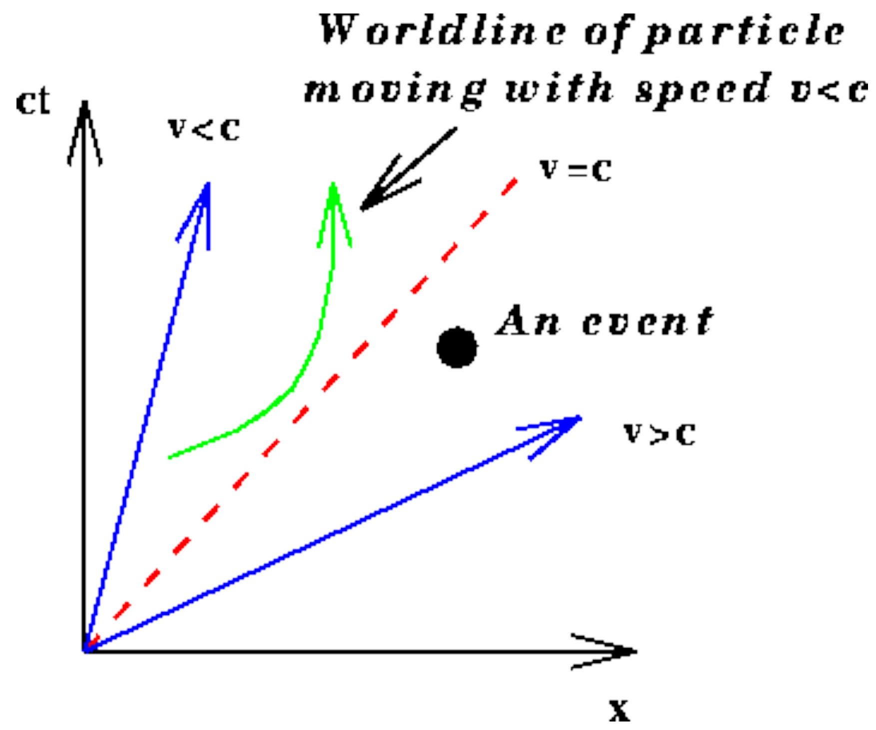
$$y' = y$$

$$z' = z$$

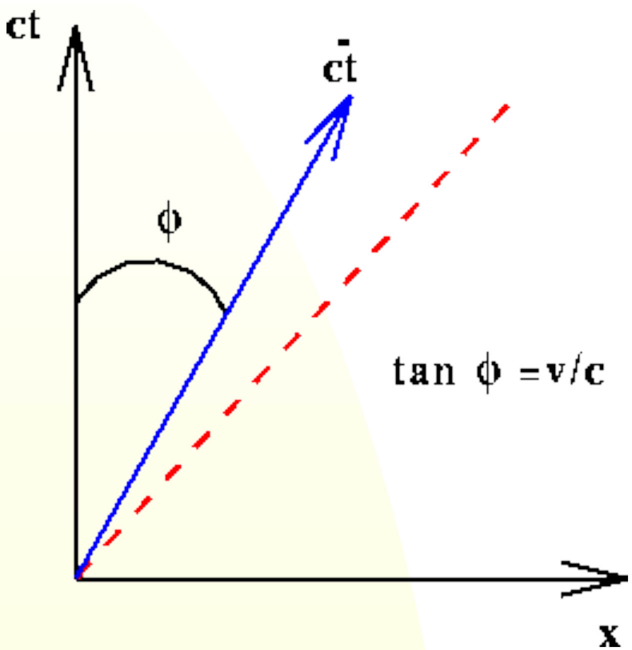
$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The reference frames coincide at t=t'=0. The point x' is moving with the primed frame.

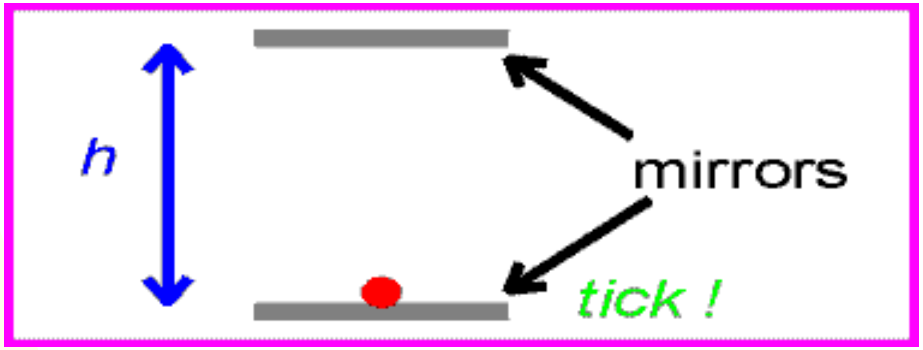
Spacetime



Worldline of particle moving with speed v<c
ct
v<c
v =c
An event
v>c
x

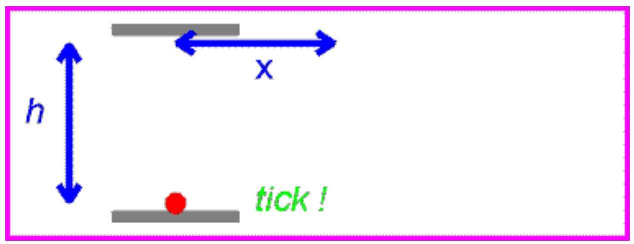

ct
ct̄
φ
tan φ =v/c
x

Time dilation



$$t_0 = h/c$$

Moving clocks run slow



$$t = \frac{\sqrt{h^2 + x^2}}{c} = \frac{\sqrt{h^2 + (vt)^2}}{c}$$

$$t = \gamma t_0 \quad , \quad \gamma \equiv \frac{1}{\sqrt{1 - v^2 / c^2}}$$

Length contraction

- Moving objects appear shorter in the dimension parallel to their velocity

$$L = \frac{L_0}{\gamma} \quad , \quad \gamma \equiv \frac{1}{\sqrt{1 - v^2 / c^2}}$$

Twins paradox

A paradox is a seemingly self-contradictory result that presents a puzzle as to its interpretation

- Consider twins separated at birth. One twin is sent traveling at 95% the speed of light around the galaxy. The second twin remains on Earth.
- After 80 years the astronaut twin returns, but according to clocks in the space ship the twin has only been gone 25 years due to time dilation. Thus one twin is physiologically 25ys old while the other is 80ys old.

- Why paradox ?

To the twin in the rocket, the first Earth-bound twin's clocks should be slower. Both twins can not be younger than the other twin. However, the relations for time dilation were derived for clocks in uniform IFRs.

The rather complex considerations of accelerating objects would shown that indeed, the astronaut twin has not aged nearly so much as his Earth-bound duplicate.

Adding velocities

- Consider two objects. The first object moves with velocity ***v*** relative to the second object, while the second object moves with velocity ***u*** with respect to an observer. In Newtonian physics the observer would say that the velocity of the first object is

.

- $V' = U + V$

- However, this would allow the observed velocity to exceed the speed of light. In fact, when relativistic effects are accounted for the expression becomes

$$v' = \frac{u + v}{1 + \frac{uv}{c^2}}$$

- One can check that this velocity never exceeds the speed of light

by working out the result when either ***u*** or ***v*** equals the speed of light. In that case, ***v'*** will be c---light speed.

- Velocity addition problems, can be stated in many ways, often with subtraction of velocities being used
- The best strategy is to always write the Newtonian expression first. Then use the relativistic expression above, assigning ***u*** and ***v*** to whatever velocities are . If the Newtonian expression has a negative velocity, one must remember to make the velocity in the numerator negative as well.

Momentum and energy

- Perhaps the most famous expression in physics is

$$E= m\, c^2$$

- This expresses the fact that an object at rest has energy

- A moving object has an energy and momentum given by
- $P = \gamma m V$
- $E = \gamma m c^2$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Note that as V approaches ***c*** that both P and E can become infinite. Thus, even though an object's velocity is constrained to be less than ***c***, the object's momentum and energy are not bounded.

Lorentz Transformation

- X’ μ =L μ γ X γ

X=(x, y, z, ict)

K=(Kx, Ky, Kz, i ω /c)

Examples for relativity

- Example #1

A muon has a lifetime of 2.2E-8 ***s*** in its own rest frame. If it travels with a speed of 0.95c, how far will it travel before it decays?

- Solution:

The distance if ***vt***, but the time is longer by a factor **gamma**.

20.1 *m*

- Example #2
- A meter stick flies by with an apparent length of 60 *cm*. What is its velocity?
- Solution:
- Starting with **g** equal to (1/0.6), solve for ***v***.
- ***v*** = 0.8c = 2.4E8 *m/s*

- Example #3
- Problem:
- Two space ships approach each other with velocities of 0.9c. According to an observer on the space ship, what is the velocity of the other ship.
- Solution:
- Use the velocity addition formula, . Both ***u*** and ***v*** are 0.9c.
- ***v'*** = 0.9944c

- Example #4
- Problem:
- a.) Two space ships travel in the same direction, with one travelling at 0.9c, the other travelling at 0.99c. What is the velocity of the faster ship according to an observer on the slower ship.

- Solution:
- Again use the velocity addition formula, with $\boldsymbol{u}$ = 0.99c and $\boldsymbol{v}$ = -0.9c.
- $\boldsymbol{v'}$ = 0.825c
- b.) What is the velocity of the slower ship according to an observer on the faster ship?
- $\boldsymbol{v'}$ = -0.825c